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# Do Higher Order Perturbative Corrections Upset $|V_{cb}|$ and $|V_{ub}|$ Determined from Semileptonic Widths?

N.G. Uraltsev

*Theoretical Physics Institute, Univ. of Minnesota, Minneapolis, MN 55455*  
 and  
*Petersburg Nuclear Physics Institute, Gatchina, St.Petersburg 188350, Russia\**

## Abstract

It is shown that large perturbative corrections found previously for semileptonic beauty and charm decays are associated with using inappropriate pole masses. The latter, in the perturbative expansion, suffer from the  $1/m_Q$  infrared renormalon which is absent in the widths, which leads to similar large corrections in  $m_Q$ . Pole masses are neither measured directly in experiment. If the widths are related to parameters determined in experiment, the overall impact of the calculated second order corrections becomes strongly suppressed and leads to less than 1% change in  $|V_{cb}|$  and  $|V_{ub}|$ . Even in charm decays the perturbative corrections appear to be very moderate in the consistent OPE-compliant treatment. The updated estimate of  $|V_{cb}|$  is given, based on recent accurate determination of  $m_b$  and  $\alpha_s(1\text{ GeV})$ . The theoretical accuracy of determination of  $|V_{ub}|$  from  $\Gamma_{sl}(b \rightarrow u)$  appears to be good as well.

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\*Permanent address

Inclusive semileptonic decays of  $B$  mesons provide at present the most accurate determination of the CKM mixing element  $|V_{cb}|$  and, if one could measure the inclusive width  $\Gamma(B \rightarrow X_u \ell \nu)$ , the value of  $|V_{ub}|$  as well. In order to reach the ultimate possible theoretical precision it is practically necessary to account for purely perturbative corrections as accurately as possible. The first order corrections are taken directly from the QED calculations for  $\mu$  decay known since 50s [1]. Recently the part of the second order corrections has been computed, which is associated with the running of the strong coupling (we will refer to such approximation below as to the BLM approach). Let us introduce the following general notations:

$$\frac{\Gamma_{\text{sl}}(m_q/m_b)}{\Gamma_{\text{sl}}^0(m_q/m_b)} = \left( 1 + a_1 \frac{\alpha_s}{\pi} + a_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots \right) \quad . \quad (1)$$

Then the result for  $\Gamma_{\text{sl}}(b \rightarrow q \ell \nu)$  [2] reads (for  $\alpha_s(m_b)$  in the  $V$  scheme [3] and  $n_f = 3$ )

$$\begin{aligned} a_1 &= -2.41 \quad , \quad a_2 = -19.7 \quad \text{at } m_q/m_b = 0 \\ a_1 &= -1.67 \quad , \quad a_2 = -8.8 \quad \text{at } m_q/m_b = 0.3 \end{aligned} \quad (2)$$

where the first line corresponds to the  $b \rightarrow u$  decays and the second one is relevant for the  $b \rightarrow c$  transitions. In Eq. (2) only the BLM part of  $a_2$  is shown. In the absence of the complete calculations we will discuss only this part to the rest of this paper without explicit reminding this reservation. It is worth noting that the values of  $a_2$  quoted in Eq. (2) correspond to the  $V$  scheme which has the direct physical meaning in the BLM calculations<sup>1</sup>. For this reason we will consistently use below the  $V$  scheme.

The above values of  $a_2$  are surprisingly large at first sight and could be thought to change significantly the theoretical estimates for the semileptonic widths, in particular in the  $b \rightarrow u$  decay, once  $(\alpha_s/\pi)^2$  terms are accounted for. It appears, however, that the actual impact of these terms is essentially smaller, because similar large  $(\alpha_s/\pi)^2$  corrections affect the values of the heavy quark masses. This ‘‘conspiracy’’ is the reflection of the dominance of the leading  $1/m_Q$  infrared renormalon which, in reality, is absent in the widths once they are expressed in terms of observable quantities [4]. The numerical situation with perturbative corrections in semileptonic beauty decays is, therefore, rather similar to the case of  $t \rightarrow b + W$  width [5].

## 1 Heavy Quark Masses and Perturbative Corrections to Widths

Heavy quark masses are not directly observed and must be determined indirectly from independent measurements. At present the most accurate determination of

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<sup>1</sup> $\overline{MS}$  scheme is unnatural here; in this scheme one would have  $a_2^{(\overline{MS})} = a_2^{(V)} + 5b/12 a_1$  where  $b = 11 - 2/3n_f \simeq 9$  is the first coefficient in the strong coupling  $\beta$ -function.

$m_b$  follows from the sum rules for  $\bar{b}b$  threshold production in  $e^+e^-$  annihilation [6]; the value of  $m_b$  there undergoes perturbative corrections as well and this must be accounted for properly.

The typical momentum scale in the analysis of moments of the spectral density in Ref. [6] is about 2 GeV, and, therefore, these sum rules are sensitive to the mass normalized at the scale  $\mu \gtrsim 1$  GeV. It does not prevent expressing the final result in terms of the “one loop pole mass”

$$m_{\text{pole}}^{(1)} \simeq m(\mu) + \frac{-dm(\mu)}{d\mu} \mu \simeq m(\mu) + c_m \frac{\alpha_s(\mu)}{\pi} \mu \quad (3)$$

as long as the one loop corrections are concerned. However, accounting for the  $\alpha_s^2$  terms shifts the corresponding value of the pole mass to the value of the “two loop pole mass” which is given in the BLM approximation by

$$m_{\text{pole}}^{(2)} \simeq m(\mu) + c_m \frac{\alpha_s(\mu)}{\pi} \mu + c_m \frac{b}{2} \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \mu = m_{\text{pole}}^{(1)} + c_m \frac{b}{2} \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \mu \quad (4)$$

Numerically this increase is as large as 150 MeV and this strongly affects the width as well. (Eqs. (3) and (4) assume that  $\mu \ll m_Q$ .)

It is more practical, both theoretically [4] and phenomenologically, to reexpress even the perturbative relation for the width in terms of the running masses normalized at the scale  $\mu$ . To the second order in  $\alpha_s$  it would require the explicit knowledge of the complete two loop scale dependence of the heavy quark mass, which is not known yet <sup>2</sup>. Being interested only in the BLM-type contribution to  $a_2$ , on the other hand, we need to account only for the terms  $\sim b(\frac{\alpha_s(\mu)}{\pi})^2$  which are easily obtained from the first order calculation. This relation becomes especially simple in the heavy quark expansion [4] which holds when  $\mu \ll m_Q$ :

$$m_{\text{pole}} \simeq m(\mu) + c_m \frac{\alpha_s(\lambda)}{\pi} \mu + c_m \frac{b}{2} \left( \frac{\alpha_s(\lambda)}{\pi} \right)^2 \left( \log \frac{\lambda}{\mu} + 1 \right) \mu + \dots \quad (5)$$

where  $\lambda$  is an arbitrary renormalization scale for  $\alpha_s$  (in the  $V$  scheme). Then one finds for the  $b \rightarrow u$  case

$$\begin{aligned} \tilde{a}_1(\mu) &= -\frac{2}{3} \left( \pi^2 - \frac{25}{4} \right) + 5 \frac{\mu}{m_b} c_m \\ \tilde{a}_2(\mu) &= a_2 + 5 \frac{\mu}{m_b} c_m \frac{b}{2} \left( \log \frac{\lambda}{\mu} + 1 \right) \quad ; \end{aligned} \quad (6)$$

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<sup>2</sup>The two loop running of  $m_Q$  is known in the dimensional regularization, which, however, is irrelevant for  $\mu \ll m_Q$ .  $m_Q(\mu)$  in naive dimensional regularization has nothing common to the properly infrared defined mass needed for performing OPE [4].

$\tilde{a}(\mu)$  refer to the perturbative coefficients utilizing running masses (for simplicity we do not change the argument of  $\alpha_s$  here). Adopting for the purpose of illustration  $\mu = 1 \text{ GeV}$  and  $c_m = \frac{4}{3}$  (see [4]), we get for  $\lambda = m_b$

$$\tilde{a}_1(\mu) \simeq -1.03 \quad , \quad \tilde{a}_2(\mu) \simeq -3.7 \quad . \quad (7)$$

The perturbative corrections appear to be of “normal” magnitude.

A similar consideration can be applied to the  $b \rightarrow c$  decays, with only minor technical complications. Here one is relatively close to the SV limit where the total width would depend only on  $m_b - m_c$  rather than on the absolute values of masses. Then the impact of using the pole masses instead of the running ones with  $\mu \ll m_c$ ,  $m_b$  is suppressed, and one gets essentially smaller values of  $a_2$  from the very beginning. In particular, if one uses the strong coupling normalized at the scale  $\sqrt{m_c m_b}$  which is appropriate for the SV kinematics [7, 8] then the value of  $a_2$  in the  $V$  scheme is

$$a_2 \simeq -4.3 \quad .$$

This coefficient, still somewhat enhanced, *per se* would lead to the increase in the value of  $|V_{cb}|$  by 2 percentage points. On the other hand, according to Ref. [9], just the increase in the input value of  $m_b$  by 150 MeV leads to the decrease of  $|V_{cb}|$  by 2.5 percentage points. Therefore when one consistently proceeds from the first order approximation to the second one, the two effects tend to strongly offset each other. We will show below that the quoted value of  $a_2$  is indeed largely due to the residual dependence of the tree level width on the absolute values of masses for actual  $m_c$  and  $m_b$  and, therefore, in reality the impact of the second order corrections is further suppressed.

To proceed from the pole masses to  $m_c(\mu)$  and  $m_b(\mu)$  one can follow the same way as for  $b \rightarrow u$ , however in this case the leading in  $\mu/m_c$  approximation used in Eqs. (3)–(5) *a priori* could be not well justified. The corrections can be readily taken into account and, as a matter of fact, appear to be small: the deviation from the linear  $\mu$  dependence is less than 7% even for  $\mu/m_c = 1$ . Let us briefly discuss the general case of arbitrary  $\mu/m_Q$ .

In the framework of the BLM approach the exact  $\mu$  dependence is given by the value of the one loop diagram evaluated with the running  $\alpha_s(k^2)$ :

$$m_Q(\mu) - m_Q(\mu') = \int_{\mu'^2}^{\mu^2} \frac{\alpha_s(k^2)}{\pi} F_m(k^2) dk^2 \quad (8)$$

where  $F_m(k^2)$  is obtained by integrating the expression for the one loop Feynman graph over the directions of the gluon momentum in the 4 dimensional Euclidean space. In fact, this quantity is conveniently expressed in terms of the one loop correction evaluated with the massive gluon propagator: assigning the gluon mass  $\nu$  one calculates, say,  $\delta m_Q^{(1)}(\nu^2)$  substituting  $1/k^2 \rightarrow 1/(k^2 + \nu^2)$  in the gluon propagator. Then  $k^2 F(k^2)$  for any observable is equal to the discontinuity of the corresponding

$\nu^2$ -dependent quantity with respect to  $\nu^2$  at the negative value of the gluon mass squared. For example, for the heavy quark mass one has

$$\frac{\alpha_s}{\pi} \cdot k^2 F_m(k^2) = -\frac{1}{2\pi i} \left( m_Q^{(1)}(\nu^2 = -k^2 + i\epsilon) - m_Q^{(1)}(\nu^2 = -k^2 - i\epsilon) \right) \quad . \quad (9)$$

Eq. (9) thus expresses the generic function  $w$  considered recently in Ref. [10] in terms of the gluon mass dependent observable which had been introduced in Refs. [11, 12, 13].

Expanding Eq. (8) in  $\alpha_s(\lambda)$  one gets

$$m_{\text{pole}} = m_Q(\mu) + \frac{\alpha_s(\lambda)}{\pi} \int_0^{\mu^2} F_m(k^2) dk^2 + \frac{b}{4} \left( \frac{\alpha_s(\lambda)}{\pi} \right)^2 \int_0^{\mu^2} F_m(k^2) \log \frac{\lambda^2}{k^2} dk^2 + \dots \quad (10)$$

(the pole mass to any finite order is defined as  $\lim_{\mu \rightarrow 0} m_Q(\mu)$ ).

At  $\nu^2 \ll m_Q^2$  one has [14, 4]  $m_Q^{(1)}(\nu^2) - m_Q^{(1)}(0) \simeq -\frac{2}{3}\alpha_s \sqrt{\nu^2}$  and, thus,  $F_m(k^2) \simeq 2/(3\sqrt{k^2})$  which yields Eqs. (3), (5) with  $c_m = 4/3$ . The correction factors to these approximate relations are easily calculated using the exact one loop expression for  $m_Q^{(1)}(\nu^2)$ :

$$m_Q^{(1)}(\nu^2) = m_Q(0) - \frac{2}{3} \frac{\alpha_s}{\pi} m_Q \left[ \int_0^1 d\alpha (1 + \alpha) \log \left( 1 + \frac{\nu^2}{m_Q^2} \frac{1 - \alpha}{\alpha^2} \right) \right] \quad . \quad (11)$$

The integral is computed straightforwardly (see, e.g. [13]) but is inessential for us. The discontinuity over  $\nu^2$  is obtained directly from Eq. (11) and takes the form

$$F_m(k^2) \equiv \frac{1}{2m_Q} \frac{\mathcal{F}_m(t)}{\sqrt{t}} = \frac{1}{3m_Q} \left( \left( 1 - \frac{t}{2} \right) \sqrt{1 + \frac{4}{t} + \frac{t}{2}} \right) \cdot \theta(t) \quad , \quad (12)$$

$$\mathcal{F}_m(t) \simeq \frac{4}{3} - \frac{t}{2} + \frac{t^{3/2}}{3} - \dots \quad , \quad t = \frac{k^2}{m_Q^2} \quad .$$

This coincides with the expressions quoted in Refs. [13, 10, 15]. The correction factors corresponding to integration of the exact function  $\mathcal{F}_m$  instead of its approximate value  $4/3$  for small  $k^2$ , differ from unity by only 0.075 and 0.05 for the first and second orders in  $\alpha_s$ , respectively, even at  $\mu = m_Q$ .

Taking all these corrections into account and using

$$\Gamma_{\text{sl}} = \frac{m_b^5 |V_{qb}|^2}{192\pi^3} z_0 \left( \frac{m_q^2}{m_b^2} \right) \left( 1 + a_1 \frac{\alpha_s}{\pi} + a_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \dots \right) \quad (13)$$

with

$$z_0(x) = 1 - 8x - 12x^2 \log x + 8x^3 - x^4$$

one gets

$$\tilde{a}_1(\mu) = -1.03 \quad , \quad \tilde{a}_2(\mu) = -3.7 \quad \text{for } b \rightarrow u$$

$$\tilde{a}_1(\mu) = -1.07 \quad , \quad \tilde{a}_2(\mu) = 0.9 \quad \text{for } b \rightarrow c \quad (14)$$

for  $\mu = 1 \text{ GeV}$ ,  $m_b = 4.8 \text{ GeV}$  and  $m_c/m_b = 0.3$  (as previously, I use the  $V$  scheme  $\alpha_s$  normalized at  $m_b$  for  $b \rightarrow u$  and at  $\sqrt{m_c m_b}$  for  $b \rightarrow c$ ). The values of the second order coefficients are moderate and  $\tilde{a}_1(\mu)$  are suppressed as compared to the case of the pole masses.

Let us emphasize that the estimates above were for illustration purposes only; to show the dominant source of the perturbative coefficients I deliberately limited myself only to the redefinition of masses while not changing the scale of  $\alpha_s$  or the *numerical* values of representative quark masses. These additional modifications are accounted for properly in the final numerical evaluation; not surprisingly, they do not change the width by any noticeable amount.

We now dwell on the definition of the renormalization point  $\mu$  for masses used above. The separation of low and high momenta can be accomplished technically in different ways leading, e.g., to somewhat different values of the coefficient  $c_m$ . The most natural choice for nonrelativistic expansion is the cutoff over the spacelike gluon momenta,  $|\vec{k}| > \mu$ , used in Ref. [16] for heavy flavor transitions in the SV kinematics. It is also most natural for the nonrelativistic  $b\bar{b}$  in the  $\Upsilon$  system. This method applied to calculation of the heavy quark mass yields the value  $c_m = 4/3$  used above. In general, in the BLM-type calculations the step-like cutoff over Euclidean four momentum  $\theta(k^2 - \mu^2)$  acts differently; however, in the case of the heavy quark mass  $\mu$ -dependence the two approaches are equivalent in the leading in  $\mu/m_Q$  approximation<sup>3</sup>.

It was found in Ref. [6], in accord with the general arguments above, that the one loop value of the  $b$  quark pole mass  $m_{\text{pole}}^{(1)}$  was strongly correlated with the obtained value of  $\alpha_s$ . On the other hand, the one loop pole mass normalized at  $1.3 \text{ GeV}$ ,

$$m_{\text{pole}}^{(1)} - \frac{4}{3} \frac{\alpha_s}{\pi} \cdot 1.3 \text{ GeV}$$

was essentially uncorrelated with  $\alpha_s$  and literally had significantly smaller range of variation [6]. Therefore, such a normalization scale can be considered as physically appropriate one; at least, it eliminates the strong correlation to the first loop approximation. The possible remaining (not enhanced) second order corrections to the mass normalized at this scale are then the part of the theoretical uncertainty of determination of the heavy quark mass in the approach of Ref. [6]. Clearly, one can, in principle, use arbitrary normalization point  $\mu$  of approximately this scale. The dependence of  $\tilde{a}_1$  and  $\tilde{a}_2$  on  $\mu$  at  $c_m = 4/3$  is illustrated in Table 1. Perturbative corrections appear to be suppressed for any reasonable choice of  $\mu$ .

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<sup>3</sup>A word of caution is worthwhile. If one uses not a Lorentz invariant cutoff then the relativistic dispersion law can be modified. For example, the effective heavy quark mass which determines the rest frame energy may start to differ from the one in the dispersion law  $E(\vec{q}) - E(0) = \vec{q}^2/2m_Q + \dots$ . It is for this reason the slightly different value of  $c_m = 16/9$  was obtained in Refs. [17, 16] considering the recoil effects in the second sum rules in the SV limit. I am grateful to M. Voloshin for the discussion of this point. In the concrete version of the BLM approach considered in the present paper this subtlety is absent.

The following remark is in order here. The value of the first order coefficient is also noticeably suppressed if one uses the running masses  $m_Q(\mu)$  above instead of the pole masses. Accordingly, the value of  $\tilde{a}_2$  is less sensitive to the particular scheme for  $\alpha_s$ . For example, even in the  $\overline{\text{MS}}$  scheme the values of  $\tilde{a}_2$  for  $\mu = 1.3 \text{ GeV}$  become  $-3.4$  and  $-1.5$  for  $b \rightarrow u$  and  $b \rightarrow c$ , respectively, i.e. small as well. Moreover, it is clear that the value of the whole perturbative factor through the second order,  $1 + \tilde{a}_1\alpha_s/\pi + \tilde{a}_2(\alpha_s/\pi)^2$  does not exhibit any noticeable dependence on the scheme chosen for  $\alpha_s$  (which is always present at some level due to the truncation of the series).

The above analysis for the  $b \rightarrow u$  case shows that practically the whole large second order contribution obtained in Ref. [2] numerically comes from the domain of gluon momenta  $\sim (0.2 \div 0.25)m_b$  and, therefore, is mainly associated with the leading  $1/m_Q$  renormalon in the pole mass which, however, is absent in the width in the proper treatment [4, 12]. Applying this observation to the semileptonic charm decays we identify that the large “nonconvergent” second order correction pointed out in [2] came, in fact, from the gluon momenta about and *below* the infrared pole in the gluon propagator; this clearly explains the observed numerical behavior. This contribution is, therefore, phenomenologically irrelevant and is explicitly excluded even from perturbative corrections in the consistent treatment. In particular, for  $\mu \simeq 400 \text{ MeV}$ , which seems to be the minimal reasonable cutoff, the perturbative corrections for charm literally become only

$$1 - 0.5 \frac{\alpha_s}{\pi} - 0.4 \left( \frac{\alpha_s}{\pi} \right)^2 \simeq 1 - 0.07 - 0.008 \simeq 0.92 \ ;$$

the second order yields a tiny contribution only. Altogether one has very moderate effect [8]. The non-BLM higher order terms are potentially more important, but most likely are by far dominated by calculable power nonperturbative [18] and unknown “exponential” terms [19].

We see that in the conventional approach, when all intermediate results are expressed in terms of the pole mass, the final result for the width is rather sensitive to the treatment of the infrared region in the Feynman integrals; already at an accuracy level of  $5 \div 10\%$  in beauty decays care must be taken to do it in a self-consistent way. On the other hand, just the physics at this scale affects the widths very little [18, 4] (as long as one keeps the high scale heavy quark masses fixed rather than the masses of observable hadrons). Therefore it is advantageous to get rid of this low energy region and, therefore, of the pole mass in the extraction of weak mixing parameters from start. It is very important from phenomenological point of view that the sum rule analysis of the  $b\bar{b}$  production [6] allows the direct determination of the high scale  $m_b(\mu)$  with  $\mu$  in the interval  $\sim (0.75 \div 1.5) \text{ GeV}$ . The theoretical accuracy there is optimal for  $\mu \approx 1 \text{ GeV}$ , in accord with the general OPE picture, where the impact of low momentum region is described by the vacuum expectation value of  $G_{\alpha\beta}^2(\mu)$  and was estimated to be negligible [6] for any reasonable normalization scale.

This fact is crucial for reliability of theoretical calculations, because the perturbative treatment in the region where  $\alpha_s$  is not small is rather involved and, as a matter of fact, is sensitive to the various approximations. For example, going beyond the BLM computations can change drastically all corrections coming from this domain, both for the pole mass and for the width expressed through it. The situation is quite different if the low momentum region is explicitly excluded. In this case the higher order terms produce small impact. This will be numerically illustrated in Sect. 2 when the result for  $|V_{qb}|$  changes only by a minute amount due to the second order corrections, when expressed in terms of  $m_Q(1\text{ GeV})$ . Only then one can count on smallness of higher order corrections, and on the moderate size of corrections due to the residual  $\alpha_s^2$  terms not captured in the BLM approximation.

We need to emphasize here two points related to the above discussion. First, in the Wilson procedure of treatment the strong coupling domain of QCD one is to cut the perturbative integrals at the scale  $\mu$  to ensure the reasonably small value of the coupling at  $\mu$  and above. On the other hand, it is not necessary *per se* to take  $\mu$  as low as possible even for purely perturbative calculations: the operator expansion can be applied to the perturbative fields as well, expressing the contribution of the low momentum region in the perturbative loop diagrams in terms of the *perturbative* matrix elements. Moreover, in such a way the contribution of this region is most simply evaluated and even computed (see, for example, papers [4, 16] where this idea was used in the heavy quark expansion); following this strategy one can easily go beyond the first loop, or with reasonable complication, even beyond the BLM approximation, which normally is difficult for complete analytical calculations. For example, the characteristic scale in  $b \rightarrow c$  is set by  $m_b - m_c$  or  $2m_c$ , which is rather large, and thus safely allows for treatment of the domain up to 1 GeV in this way.

As an example, we can consider the expansion of the pole mass as the lowest eigenvalue of the rest frame Hamiltonian [16] obtained in  $1/m_Q$  approach to some order  $k$  in perturbation theory:

$$m_Q^{\text{pole}} - m_Q(\mu) = \sum_{n=0}^{\infty} \lambda_n(\mu) \frac{1}{m_Q^n(\mu)} = \bar{\Lambda}(\mu) + \frac{\mu_\pi^2(\mu)}{2m_Q(\mu)} + \dots \quad (15)$$

where the series runs in inverse powers of  $m_Q(\mu)$ . According to Eq. (15), the perturbative matrix elements  $\lambda_n(\mu)$  are given in the BLM approximation by the corresponding coefficients of the expansion of  $\mathcal{F}_m(t)$  (it is defined in Eq. (12)) at small  $t$ : for

$$\mathcal{F}_m(t) = \sum_{n=0}^{\infty} c_n t^{\frac{n}{2}}$$

one has

$$\begin{aligned} \lambda_n(\mu) &= c_n \frac{\alpha_s(\mu)}{\pi} \sum_{l=0}^k \int_0^\mu d\rho \rho^n \left( \frac{b\alpha_s(\mu)}{2\pi} \right)^l \log^l \frac{\mu}{\rho} = \\ &= \frac{c_n}{n+1} \frac{\alpha_s(\mu)}{\pi} \mu^{n+1} \sum_{l=0}^k \left( \frac{b\alpha_s(\mu)}{2(n+1)\pi} \right)^l l! . \end{aligned} \quad (16)$$



For  $n = 0$  we have  $\lambda_0(\mu) = \bar{\Lambda}(\mu)$  and, therefore, its  $\mu$ -dependence is indeed given by Eq. (5). Turning to the next term in  $1/m_Q$ ,  $n = 1$ , we see that  $\mathcal{F}_m(t)$  does not contain the term  $\sim t^{1/2}$ , i.e.  $c_1 = 0$ . It means that the matrix element of the kinetic operator does not receive the perturbative contribution in the BLM approximation to any order in  $(b\alpha_s/\pi)$  when the cutoff is introduced in this particular way (this fact was pointed out in the somewhat different context in Ref. [12]). Literally the same technique can be applied to other quantities, not only  $m_Q^{\text{pole}}$ , as well.

The second comment concerns the concrete way of eliminating the low energy momentum region implicitly implied in our reasoning. Clearly, in general it does not reduce only to using the running quark masses: even after expressing the width in terms of  $m_Q(\mu)$  integration in Feynman graphs determining the width runs, strictly speaking, over all momenta starting  $k^2 = 0$ . Therefore formally an additional subtraction of the remaining contribution below  $\mu^2$  is necessary. It is easy to see using the above arguments, however, that in this particular case the situation is rather simple: these extra terms can be neglected. Indeed, we can use the operator relation for the width [18, 4]

$$\begin{aligned} \Gamma_{\text{sl}} \propto m_b^5(\mu) \cdot z \left( \frac{m_c^2(\mu)}{m_b^2(\mu)} \right) \left[ a_{\text{pert}}^{(0)} \left( (m_c^2/m_b^2); \mu \right) \cdot \left( 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \right) - \right. \\ \left. - a_{\text{pert}}^G \left( (m_c^2/m_b^2); \mu \right) \cdot \frac{\mu_G^2}{m_b^2} + \dots \right] \end{aligned} \quad (17)$$

where  $\mu_\pi^2$  and  $\mu_G^2$  are the expectation values of the kinetic and chromomagnetic operators normalized at point  $\mu$ . The chromomagnetic operator does not mix with the unit one due to the different spin structure, and therefore its perturbative matrix element over heavy quark vanishes. As was mentioned just above, in the leading BLM approximation the perturbative value of  $\mu_\pi^2$  vanishes as well if one introduces the cutoff in the way adopted here. Thus one immediately concludes that the contribution of the infrared region to the perturbative coefficient of the leading operator in Eq. (17),  $a_{\text{pert}}^{(0)}$ , is suppressed by at least the third power of  $\mu/m_Q$  to any order in  $\alpha_s$  in the BLM approximation<sup>4</sup>. The BLM-type calculations of  $a_{\text{pert}}^{(0)}(\mu)$  are then very infrared stable.

We see that the contribution of the domain below  $\mu$  to the perturbative correction factor expressed in terms of  $m_Q(\mu)$  is given by the corresponding perturbative contribution to the matrix elements of higher dimension operators only whose corrections to width scale like  $1/m_Q^3$  and higher powers of the inverse mass. On the other hand, nonperturbative corrections in the width are so far accounted for only through terms  $\sim 1/m_Q^2$ . Thus as long as at the scale  $\mu \simeq 1 \text{ GeV}$  the perturbative part of matrix elements is dominated by the condensate-like effects, the neglect

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<sup>4</sup>This exactly corresponds to the result of Ref. [12] which extended the statement of Ref. [4] about the absence of terms  $\sim 1/m_Q$  through terms  $\sim 1/m_Q^2$ ; however, the latter seems to be a feature of the BLM approximation only.

of the remaining perturbative corrections from the domain  $k^2 < \mu^2$  is not only legitimate, but rather necessary.

Let us note that in the exclusive  $B \rightarrow D^{(*)}$  transition the dependence of the zero recoil amplitude on the masses enters only via the perturbative (and also nonperturbative) corrections. Strictly speaking, even here one should have used the running masses to calculate, say, the perturbative factors  $\eta_{A,V}$ . Then, for example, to the second order in  $\alpha_s$  one would have

$$\eta_A \rightarrow \eta_A + c_m \frac{\alpha_s}{\pi} \frac{\mu}{m_c} y(1-y) \frac{d\eta_A}{dy} \simeq \eta_A - c_m \left( \frac{\alpha_s}{\pi} \right)^2 \frac{\mu}{m_c} \left( 1 + y + \frac{2y}{1-y} \log y \right) \quad (18)$$

$$y = m_c/m_b \quad , \quad \mu \ll m_c \quad .$$

The effect, however, does not appear in the BLM approximation; numerically it is insignificant compared to nonperturbative corrections<sup>5</sup>. The peculiarity of the determination of  $|V_{qb}|$  from the total semileptonic width is that the latter in the tree approximation is proportional to the power of the quark masses, and therefore it does require the accurate treatment of the heavy quark masses.

## 2 Implications for Determination of $|V_{cb}|$ and $|V_{ub}|$

Let us briefly discuss the phenomenological implications. Eliminating the spurious  $1/m_Q$  renormalon which, in fact, is not present in the relation between different observables in QCD [4] by using the running quark masses instead of the pole masses, makes the perturbative corrections essentially smaller. Now we would get for  $m_c/m_b = 0.3$  and  $m_b = 4.8 \text{ GeV}$

$$\Gamma_{\text{sl}}(b \rightarrow u) \simeq \frac{\tilde{m}_b^5 |V_{ub}|^2}{192\pi^3} \left( 1 - 0.62 \frac{\alpha_s(m_b)}{\pi} - 1.1 \left( \frac{\alpha_s}{\pi} \right)^2 \right)$$

$$\Gamma_{\text{sl}}(b \rightarrow c) \simeq \frac{\tilde{m}_b^5 |V_{cb}|^2}{192\pi^3} \left( 1 - 0.86 \frac{\alpha_s(\sqrt{m_c m_b})}{\pi} + 1.7 \left( \frac{\alpha_s}{\pi} \right)^2 \right) \quad (19)$$

$$\tilde{m}_b = m_b(1.3 \text{ GeV}) \quad , \quad \tilde{m}_c = m_c(1.3 \text{ GeV}) \quad .$$

Addressing the CKM mixing parameters  $V_{ub}$  and  $V_{cb}$ , one needs to consider the square root of the widths, and in particular the corresponding perturbative factors

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<sup>5</sup>From the theoretical perspective, however, this modification is profound: it represents the only linear in  $1/m_Q$  correction to  $\eta_A$  as it is defined in HQET. To phrase it differently: unless one uses the running masses, i.e. introduces the corrections similar to those in Eq. (18), the known statements about the absence of corrections linear in  $1/m_Q$  to the zero recoil amplitudes from the low energy domain, are incorrect. (For the most general justification of the above conclusion see [16], part 7.) The last term in Eq. (18) thus represents the counter-example to the Luke's theorem as it is formulated in HQET, see, e.g. [20]; this effect appears only beyond the BLM approximation, at the order  $\alpha_s^2$  and has not been noticed so far.

$\eta_\Gamma = (\Gamma_{\text{sl}}^{\text{pert}}/\Gamma_{\text{sl}}^0)^{1/2}$  which are directly related to extraction of  $|V_{qb}|$ . In terms of  $\tilde{m}$  they then look as follows:

$$\begin{aligned}\tilde{\eta}_\Gamma &\simeq \left(1 - 0.31 \frac{\alpha_s(m_b)}{\pi} - 0.6 \left(\frac{\alpha_s}{\pi}\right)^2\right) \quad \text{for } b \rightarrow u \\ \tilde{\eta}_\Gamma &\simeq \left(1 - 0.43 \frac{\alpha_s(\sqrt{m_c m_b})}{\pi} + 0.8 \left(\frac{\alpha_s}{\pi}\right)^2\right) \quad \text{for } b \rightarrow c\end{aligned}\quad (20)$$

These approximate expressions show the real sensitivity of extraction of  $|V_{ub}|$  and  $|V_{cb}|$  to the perturbative effects. For example, the impact of the second order BLM correction is smallish. Moreover, because now the values of the heavy quark masses are essentially uncorrelated with  $\alpha_s$ , the first order coefficient indicates the actual uncertainty in  $|V_{cb}|$  associated with the precise value of  $\alpha_s$ , which thus appears to be small and does not exceed the one occurring for the exclusive  $B \rightarrow D^*$  zero recoil amplitude.

Using the most recent evaluation of  $m_b$  we can now obtain the updated determination of  $|V_{cb}|$  and the expression for  $|V_{ub}|$ ; although they are not really affected by the second order BLM-type corrections, we will use the calculated terms [2] literally to obtain the ‘central’ theoretical values. Two clarifying comments are in order beforehand.

The above numerical values of the coefficients were shown for illustrative purposes only; in fact, the ratio  $m_c/m_b$  even for the one loop pole masses appears to be less than 0.3 when the  $1/m_Q$  expansion relations (see Eq. (21) below) are respected; this ratio decreases further for the running masses. In our numerical analysis we compute the values of the coefficients anew for proper values of  $m_b$  and  $m_c/m_b$ .

Second comment concerns the value of  $m_c$ . In principle, it can be accurately determined independently from charmonium sum rules, or from the semileptonic  $b \rightarrow c$  spectrum itself [21, 22]. At present, however, the most accurate estimate follows from the relation

$$m_b - m_c = \frac{3M_{B^*} + M_B}{4} - \frac{3M_{D^*} + M_D}{4} + \frac{\mu_\pi^2}{2} \left( \frac{1}{m_c} - \frac{1}{m_b} \right) + \mathcal{O} \left( \frac{1}{m_Q^2} \right) \quad (21)$$

which has been used for accurate determination of  $|V_{cb}|$  in Refs. [9, 16]. This relation is formally valid for masses normalized at any legitimate point  $\mu \gg \Lambda_{\text{QCD}}$ , and for our purposes it would be most convenient to take  $\mu$  directly the same as used in Sect. 1,  $\mu \sim 1 \text{ GeV}$ . Strictly speaking, the result depends to some extent on the particular choice of  $\mu$  due to terms  $\sim (\alpha_s/\pi) \mu^3/m_Q^2$  (and higher in  $\alpha_s/\pi$  and/or  $\mu/m_Q$ ) contributing to Eq. (21) in the form of the perturbative pieces of expectation values of higher dimension operators. In particular, one may be concerned with the terms  $\sim \alpha_s \mu^3/m_c^2$  for large values of  $\mu \simeq m_c$ . In fact, this effect is easily controlled to all orders in the framework of the BLM approximation,

$$m_b(\mu') - m_c(\mu') = m_b(\mu) - m_c(\mu) + \int_{\mu^2}^{\mu'^2} dk \frac{\alpha_s(k)}{\pi} \left( \mathcal{F}_m \left( \frac{k^2}{m_c^2} \right) - \mathcal{F}_m \left( \frac{k^2}{m_b^2} \right) \right) \simeq$$

$$\begin{aligned} \simeq & m_b(\mu) - m_c(\mu) + \frac{\alpha_s(\lambda)}{\pi} \int_{\mu^2}^{\mu'^2} dk \left( \mathcal{F}_m \left( \frac{k^2}{m_c^2} \right) - \mathcal{F}_m \left( \frac{k^2}{m_b^2} \right) \right) + \\ & + \frac{b}{2} \left( \frac{\alpha_s(\lambda)}{\pi} \right)^2 \int_{\mu^2}^{\mu'^2} dk \log \frac{\lambda}{k} \left( \mathcal{F}_m \left( \frac{k^2}{m_c^2} \right) - \mathcal{F}_m \left( \frac{k^2}{m_b^2} \right) \right) + \dots \end{aligned} \quad (22)$$

The variation of the mass difference does not exceed 20 MeV even when  $\mu'$  varies up to 1.4 GeV. This dependence on the normalization point is less than the error associated with the existing uncertainty in  $\mu_\pi^2$  and can be discarded<sup>6</sup>. For the sake of definiteness we use the relation (21) at the scale 0.5 GeV; this literally decreases the value of  $m_b(\mu) - m_c(\mu)$  obtained without radiative corrections by only a few MeV.

For our numerical estimates we use, as the central value, the strong coupling determined from the sum rules for  $b\bar{b}$  production [6] as well:

$$\alpha_s^{\overline{\text{MS}}}(1 \text{ GeV}) = \alpha_s^V(2.3 \text{ GeV}) = 0.336 \pm 0.011 \quad (23)$$

and the value [6]

$$m_b^* \simeq m_b(1.3 \text{ GeV}) \simeq m_b^{\text{pole}} - 0.56 \alpha_s^V(2.3 \text{ GeV}) \cdot 1 \text{ GeV} = 4.639 \text{ GeV} ; \quad (24)$$

in view of the numerical close proximity of the SV scale  $\sqrt{m_c m_b}$  to the above  $V$  scheme scale we expressed the perturbative corrections in terms of  $\alpha_s^V(2.3 \text{ GeV})$  in the case of  $b \rightarrow c$  decays irrespective of the exact values of the running quark masses, and for  $b \rightarrow u$  decays use  $\alpha_s^V(4.8 \text{ GeV})$ ; the second order coefficients are then adjusted appropriately. Again, this does not lead to any noticeable variation as compared to different choices.

We then obtain for  $\mu_\pi^2 = 0.5 \text{ GeV}^2$  using the normalization scale  $\mu = 1 \text{ GeV}$ :

$$|V_{ub}| = 0.00458 \left( \frac{\text{Br}(B \rightarrow X_u \ell \nu)}{0.002} \right)^{\frac{1}{2}} \left( \frac{1.6 \text{ ps}}{\tau_B} \right)^{\frac{1}{2}}$$

and

$$|V_{cb}| = 0.0408 \left( \frac{\text{Br}(B \rightarrow X_c \ell \nu)}{0.105} \right)^{\frac{1}{2}} \left( \frac{1.6 \text{ ps}}{\tau_B} \right)^{\frac{1}{2}} \quad (25)$$

It is important to emphasize that neither of the numbers above depend essentially on the exact value of the scale  $\mu$ : varying it from 0 (i.e. using the “two loop pole mass”) to 1.35 GeV literally changes the value of  $|V_{cb}|$  by only  $-1.2$  percentage points. Using the running masses is only instructive in showing explicitly the small effect of higher order corrections; it is not mandatory in practice as long as one uses the same approximations in the theoretical expression for the widths and for

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<sup>6</sup>The suppression of corrections to the mass difference reflects the fact that they start with terms  $\sim (\mu/m_Q)^3$ ; this follows from the absence of the perturbative contribution to  $\mu_\pi^2$  in the BLM approximation.

determination of quark masses [16]. The dependence is more pronounced in the case of  $b \rightarrow u$ , but even here  $|V_{ub}|$  increases by only 2 percentage points when  $\mu$  is descended from 2 GeV down to 0.5 GeV. Let us finally note that discarding the second order perturbative terms  $\sim b(\alpha_s/\pi)^2$  altogether would literally lead to the explicit coefficients 0.00460 and 0.0410 in the above equations, respectively, provided one uses the same running masses normalized at 1 GeV. The change is less than a half of percentage point <sup>7</sup>!

Turning to the estimate of the actual theoretical accuracy of extracting  $|V_{cb}|$  we note that the dominant uncertainty comes from the value of  $\mu_\pi^2$ : varying it by  $\pm 0.1 \text{ GeV}^2$  changes  $|V_{cb}|$  by the factor  $(1 \mp 0.013)$  which, in fact, comes mainly from the related variation of  $m_b - m_c$ . The dependence on the exact value of  $m_b^*$  is rather weak: changing it by  $\pm 30 \text{ MeV}$  leads to the shift in  $|V_{cb}|$  by  $\mp 0.6$  percentage points.

The main uncertainty in  $|V_{cb}|$  is associated with the exact value of  $m_b - m_c$ . Changing it by  $\pm 30 \text{ MeV}$  as compared to the one given by Eq. (21) leads to the factor  $(1 \mp 0.011)$ . The relation (21) is exact to the order  $1/m_Q^2$ , however it may well be affected at the level of 20 MeV by terms  $\sim 1/m_c^3$ . Therefore, at a percent level of accuracy one needs to estimate  $1/m_Q^3$  terms in the heavy quark expansion if relies on this mass relation for fixing  $m_c$ . On the other hand, the value of  $m_b - m_c$  can be accurately determined from the semileptonic spectrum itself not appealing to the expansion in  $1/m_c$ ; then the nonperturbative corrections calculated [21] through terms  $1/m_b^2$  are sufficient to determine the mass difference with the necessary accuracy [22], and even when  $m_c$  is not large. Combining the two methods one can, therefore, eliminate this source of uncertainty in  $|V_{cb}|$  at a percent level in the both limits of large and small  $m_c$ , i.e. have the determination of  $|V_{cb}|$  limited, on theoretical side, only by terms  $\propto 1/m_b^3$ .

Finally, the dependence on the exact value of  $\alpha_s$  is also rather moderate: changing  $\alpha_s^{\overline{\text{MS}}}(1 \text{ GeV})$  by  $\pm 0.02$  emerges in the variation of  $|V_{cb}|$  by  $\pm 0.6$  percentage points only.

Even better stability would hold for  $|V_{ub}|$ : the dependence on  $\mu_\pi^2$  is very weak here,  $\pm 0.12$  percentage points for every  $\pm 0.1 \text{ GeV}^2$  in the latter. The similar change in  $m_b^*$  by  $\pm 30 \text{ MeV}$  generates the variation by the factor  $(1 \mp 0.016)$  and the uncertainty of  $\pm 0.02$  in  $\alpha_s^{\overline{\text{MS}}}(1 \text{ GeV})$  translates into the factor  $(1 \pm 0.009)$ .

The fit error bars quoted in Ref. [6] are

$$\delta m_b^* = \pm 0.002 \text{ GeV} \quad , \quad \delta \alpha_s^{\overline{\text{MS}}}(1 \text{ GeV}) = \pm 0.011. \quad (26)$$

If used literally they would lead only to the negligible error in  $|V_{cb}|$  less than 0.3%. It is clear, however, that for our purposes it cannot be taken at face value, even leaving aside the presently unknown value of  $\mu_\pi^2$ . There are other sources of  $(\alpha_s/\pi)^2$

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<sup>7</sup>Comparing with the previous estimate [9], the central value of  $|V_{cb}|$  obtained there for the same input parameters  $\Gamma_{\text{sl}}(b \rightarrow c)$  and  $\mu_\pi^2$  as adopted in the present paper, reads  $|V_{cb}| = 0.0405$ . In this respect it is worth clarifying that the numerical estimate in Ref. [9], Eq.(25), was obtained using the  $V$  scheme coupling corresponding to the  $\overline{\text{MS}}$  one quoted in [9]; the latter was 15% larger than the central value adopted in the present paper.

perturbative corrections, both in the inclusive width and in the determination of  $m_b^*$  in Eq. (24) which are probably more important. Of course, the above quoted error in  $m_b$  is not relevant here as well, because  $m_b^*$  cannot be viewed as the exact value of the running mass at a known scale. To be on the conservative side we feel necessary, before the dedicated analysis [23] for the running mass is completed, assign much larger uncertainty to  $m_b(1\text{ GeV})$ ,  $\delta m_b \approx 50\text{ MeV}$  (the uncertainty in this running mass in the analysis of sum rules for  $b\bar{b}$  production seems to be about  $20 \div 30\text{ MeV}$  [23]), and allow for the uncertainty in  $a_2$  for the inclusive widths of at least  $\pm(3 \div 5)$  as long as the complete two loop calculations for the width are not available. Adding these theoretical error bars one ends up with the current theoretical accuracy in  $|V_{cb}|$  of about  $(3 \div 3.5)\%$  as a rather conservative estimate, provided  $\mu_\pi^2$  is known. Anyway, it is quite possible that the theoretical precision cannot be reliably pushed below a percentage level due to potential preasymptotic corrections given by “exponential” terms limiting the applicability of duality for semileptonic decays of actual  $b$  hadrons [24].

The similar theoretical accuracy of the hypothetical determination of  $|V_{ub}|$  from the inclusive width  $\Gamma(B \rightarrow X_u \ell \nu)$  might seem to be extraordinary good, better than 3%. In fact, rather sizable effects here may, in principle, come from terms  $\sim 1/m_b^3$ , in particular due to generic Weak Annihilation processes [25] possible in the KM suppressed semileptonic decays, which are expressed in terms of the expectation values of local four fermion operators [26, 27, 25, 19]; their effect can be numerically enhanced here. To some extent it can be controlled by studying semileptonic KM suppressed decays separately for charged and neutral  $B$  mesons, and in particular in the end point region where the effect mainly originates from [25]. Therefore the determination of  $|V_{ub}|$  based only on the high energy part of the lepton spectrum can undergo strong higher order nonperturbative corrections that are not well known yet. If the total  $b \rightarrow u$  semileptonic width were accurately known then the uncertainty in  $|V_{ub}|$  would not exceed 5% level.

### 3 Conclusions

We have pointed out in this paper that the large (in particular for the  $b \rightarrow u$  channel) second order perturbative coefficients for the inclusive semileptonic widths of beauty particles are mainly associated with the similar contributions to the pole masses of heavy quarks when the widths are expressed in terms of  $m_Q^{\text{pole}}$ . The corrections become small if one uses theoretically well defined running masses normalized at the scale about 1 GeV; moreover, only these masses can and, as a matter of fact, are determined from experiment with necessary accuracy. In particular, the systematic shift in the values of  $|V_{cb}|$  and  $|V_{ub}|$  expressed in terms of the corresponding semileptonic widths, constitutes less than one percentage point when proceeding from the first order perturbative expressions to the ones where the  $\alpha_s^2$  corrections are calculated using the BLM approximation, provided the value of  $m_b(1\text{ GeV})$  is fixed. Thus

the perturbative corrections, as well as nonperturbative effects, seem to be under good control at the level corresponding to a percent relative accuracy in  $|V_{qb}|$ .

Even in the semileptonic decays of charm the consistent OPE treatment leads to the smallness of perturbative corrections: the large negative value of perturbative corrections found in Refs. [2] in fact came from the momentum region near and below the infrared pole in the strong coupling. This contribution is therefore numerically irrelevant and is to be excluded from the calculations applied to charm; it is completely accounted for by nonperturbative effects within Wilson OPE. For this reason we cannot consider convincing the conjecture stated in Refs. [2] that the apparent numerical discrepancy of experimental semileptonic width of  $D$  mesons with theoretical expectations is associated with the uncontrollable nature of the perturbative series. The possibility to witness sizable violations of duality in this case [19] seems to be more probable.

The theoretical accuracy of calculations of total semileptonic widths of  $b$  particles appears to be very good when the input from the analysis of the  $b\bar{b}$  threshold domain [6] is used. One has, as the central values, at  $\mu_\pi^2 = 0.5 \text{ GeV}^2$

$$\begin{aligned} |V_{ub}| &= 0.00458 \left( \frac{\text{Br}(B \rightarrow X_u \ell \nu)}{0.002} \right)^{\frac{1}{2}} \left( \frac{1.6 \text{ ps}}{\tau_B} \right)^{\frac{1}{2}}, \\ |V_{cb}| &= 0.0408 \left( \frac{\text{Br}(B \rightarrow X_c \ell \nu)}{0.105} \right)^{\frac{1}{2}} \left( \frac{1.6 \text{ ps}}{\tau_B} \right)^{\frac{1}{2}}. \end{aligned} \quad (27)$$

The main uncertainty in  $|V_{cb}|$  at present comes from the exact value of  $\mu_\pi^2$ :

$$|V_{cb}| \propto \left( 1 - 0.013 \frac{(\mu_\pi^2 - 0.5 \text{ GeV}^2)}{0.1 \text{ GeV}^2} \right) \cdot \left( 1 - 0.006 \frac{\delta m_b^*}{30 \text{ MeV}} \right). \quad (28)$$

The conservative estimate of other uncertainties associated, in particular, with not yet calculated part of the second order perturbative corrections, is about 3 percentage points. The above expressions for  $|V_{cb}|$  practically do not differ from the previous estimate obtained in Ref. [9].

The theoretical accuracy of calculating  $\Gamma_{\text{sl}}(b \rightarrow u)$  is even better; there is no strong dependence on  $\mu_\pi^2$  here, and the precision is better than 5% of the equivalent relative variation in  $|V_{ub}|$ . The uncertainty associated with the mass of  $b$  quark does not exceed  $2 \div 3$  percentage points and is not dominant.

The analysis made in this paper suggests also that the impact of the third and higher order perturbative corrections, which can be computed straightforwardly within the BLM approximation [11, 13, 15], is to be small and under good theoretical control provided one treats the quark masses in the consistent way. Namely, if the pole mass for  $m_b$  is used, its numerical value must be calculated in exactly the same approximation as applied to the calculation of the width. For example, one can impose the constraint that the one (or two) loop pole mass is not changed by the higher order corrections. More simple, and theoretically appropriate, approach is to

express the corrections in terms of the running mass at the scale around 1 GeV, which can be even better determined from experiment. (The usual technical problem with the precise definition of normalization point is absent in the BLM approximation.) The BLM corrections can then effectively sum the potentially dominant terms coming from *high* momenta by taking into account running of  $\alpha_s$ . Because the coupling in this region is relatively small, the running is not sharp and is well approximated already by the second term in the expansion of  $\alpha_s$ . For this reason one expects numerically small impact of the BLM corrections beyond  $b(\alpha_s/\pi)^2$  terms, and the obtained series must be under good numerical control. On the contrary, the effect of the low momentum physics is to be computed by means of the Wilson OPE rather than summing up perturbative series unstable in infrared.

*Note added:* When this paper was in progress I was informed by V. Braun about computations of the effects of higher order perturbative corrections for semileptonic widths made within the technique of papers [13, 15], Ref. [28]. The preliminary results reported agreed with the expectations that the actual impact of higher order corrections is smallish when the proper corrections in the quark masses are introduced.

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	$b \rightarrow c$	$b \rightarrow c$	$b \rightarrow u$	$b \rightarrow u$
$\mu, \text{GeV}$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{a}_1$	$\tilde{a}_2$
0.3	-1.50	-1.9	-2.00	-12.6
0.5	-1.38	-0.9	-1.72	-9.5
0.75	-1.23	0.1	-1.38	-6.3
1.0	-1.07	0.9	-1.03	-3.7
1.25	-0.89	1.6	-0.70	-1.5
1.5	-0.71	2.1	-0.35	0.4
2.0	-0.32	2.8	0.32	3.5

Table 1: Dependence of the perturbative coefficients  $\tilde{a}_1$  and  $\tilde{a}_2$  on the scale  $\mu$  for  $m_b = 4.8 \text{ GeV}$  and  $m_c/m_b = 0.3$ . The strong coupling  $\alpha_s$  is assumed to be defined in the  $V$  scheme and normalized at  $\sqrt{m_c m_b}$  for  $b \rightarrow c$  and at  $m_b$  for  $b \rightarrow u$ .